

Modelling shoaling and breaking waves on a mild sloping beach

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The mathematical modelling of coastal wave is a quite challenging issue since it is difficult to describe in the same model the dispersive effects in the shoaling zone and the energy dissipation of breakers in the surf zone. In this study we propose a new model for waves propagation over a mild sloping topography. Shearing and turbulence effects in breaking waves are taken into account by a third variable called enstrophy. With Teshukov's hypothesis of weakly sheared flows, the system is closed and features depth-averaged balance equations for mass, momentum and energy. In the absence of enstrophy, the system reduces to the equations of Green-Naghdi. Enstrophy production is handled with a turbulent viscosity hypothesis and enstrophy dissipation is governed by an empirical law. Since the model is dispersive and not hyperbolic, the enstrophy equation can replace conveniently the energy equation for the numerical resolution. The equations were numerically solved with the strategy of Le Métayer et al. (2010). The scheme is rewritten for the new variables and allows us to use a hybrid method which consist in the resolution of a hyperbolic system by a Godunov-type method and an elliptic equation. The non-dissipative part of the model possesses a solitary wave solution which is confirmed numerically. The numerical simulations were successfully compared to the experimental data of Hsiao et al. (2008).